

EXPRESSIONS & OPERATIONS

Real Numbers, Expressions & Models

Algebra • Unit 1 • Topics 1.1–1.6

Goal: represent, simplify, evaluate, and model with real numbers and algebraic expressions.

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1.1 Properties of Real Numbers

Real numbers are organized in nested sets with shared properties.

1 NATURAL

Counting numbers: 1, 2, 3, ...

2 WHOLE

Natural numbers together with 0.

3 INTEGER

Whole numbers and their opposites.

4 RATIONAL

Can be written a/b ($b \neq 0$); decimals terminate or repeat.

5 IRRATIONAL

A real number whose decimal never terminates or repeats.

6 REAL

All rational and irrational numbers on the number line.

Please give some examples in each of these “sets”.

1.1 Properties of Real Numbers

Properties let us rewrite expressions without changing their value.

1

IDENTITY

0 and 1 preserve values;
opposites undo operations.

2

COMMUTATIVE

Order can change for addition
or multiplication.

3

ASSOCIATIVE

Grouping can change for
addition or multiplication.

4

DISTRIBUTIVE

Multiply $a(b + c)$ to get
 $ab + ac$.

5

CLOSURE

“Real + real” and “real $\times$ real”
stay real.

Which property is this?

$$a + b = b + a$$

1.1 Properties of Real Numbers

Classify $-\sqrt{49}$ in every real-number set to which it belongs.

1.1 Properties of Real Numbers — Worked Solution

Classify $-\sqrt{49}$ (*square root sign*) in every real-number set to which it belongs.

SOLUTION

$$-\sqrt{49} = -7$$

-7 is an integer.

$-7 = -7/1$, so it is rational.

Answer: integer, rational, and real.

It is not whole or natural.

1.1 Properties of Real Numbers

Which property justifies $8 + (-8) = 0$?

1.1 Properties of Real Numbers — Worked Solution

Which property justifies $8 + (-8) = 0$?

SOLUTION

A number and its opposite add to 0.

For any real number a :

$$a + (-a) = 0$$

Answer: the additive inverse property.

What is the inverse property for multiplication?

1.2 Order of Operations

Follow a consistent sequence so every expression has one value.

1

GROUPING

Simplify parentheses, brackets, and fraction bars first.

2

EXPONENTS

Evaluate powers before other operations.

3

LEFT TO RIGHT

Work multiply/divide, then add/subtract left to right.

4

FRACTION BAR

Groups the entire numerator and denominator.

5

NEGATIVE SIGNS

Use parentheses when a negative is squared.

Use “PEMDAS”

Parenthesis - **E**xponent - **M**ultiply/**D**ivide - **A**dd/**S**ubtract

1.2 Order of Operations

Evaluate: $3 + 2(5^2 - 7)$

1.2 Order of Operations — Worked Solution

Evaluate: $3 + 2(5^2 - 7)$

SOLUTION

$$3 + 2(5^2 - 7)$$

$$3 + 2(25 - 7)$$

$$3 + 2(18)$$

$$3 + 36$$

$$39$$

$$5^2 = 25$$

$$25 - 7 = 18$$

$$2 \times 18 = 36$$

Add 3

Answer: 39

1.2 Order of Operations

Evaluate: $48 \div 6(2 + 2)$

1.2 Order of Operations — Worked Solution

Evaluate: $48 \div 6(2 + 2)$

SOLUTION

$$48 \div 6(2 + 2)$$

$$48 \div 6(4)$$

$$8 \times 4$$

$$32$$

$$2 + 2 = 4$$

$$48 \div 6 = 8$$

Work left to right, multiply by 4

Answer: 32

Multiplication and Division have equal priority, so work left to right.

1.2 Order of Operations

Evaluate: $-3^2 + (-3)^2$

1.2 Order of Operations — Worked Solution

Evaluate: $-3^2 + (-3)^2$

SOLUTION

$$-3^2 + (-3)^2$$

$$-(3^2) + (-3)(-3)$$

$$-9 + 9$$

$$0$$

Exponent before sign

$$3^2 = 9$$

$$(-3)^2 = 9$$

$$\text{Combine: } -9 + 9 = 0$$

Answer: 0

Parentheses determine whether the negative sign is squared.

1.3 Simplifying Expressions

Like terms have identical variable parts and exponents.

1 IDENTIFY

$3x$ and $-8x$ match;
 $3x$ and $3x^2$ do not.

2 COMBINE

Add or subtract coefficients;
keep variables unchanged.

3 CONSTANTS

Numbers without variables
combine with constants.

4 DISTRIBUTE

Remove grouping symbols
before combining.

5 STANDARD FORM

Write terms in descending
powers.

1.3 Simplifying Expressions

Simplify: $7x - 3 + 4x + 9$

1.3 Simplifying Expressions — Worked Solution

Simplify: $7x - 3 + 4x + 9$

SOLUTION

$$\begin{aligned} &7x - 3 + 4x + 9 \\ &(7x + 4x) + (-3 + 9) \\ &11x + 6 \end{aligned}$$

Answer: $11x + 6$

1.3 Simplifying Expressions

Simplify: $3(2y - 5) - 4(y + 1)$

1.3 Simplifying Expressions — Worked Solution

Simplify: $3(2y - 5) - 4(y + 1)$

SOLUTION

$$\begin{aligned} &3(2y - 5) - 4(y + 1) \\ &6y - 15 - 4y - 4 \\ &(6y - 4y) + (-15 - 4) \\ &2y - 19 \end{aligned}$$

Answer: $2y - 19$

1.3 Simplifying Expressions

Simplify: $5a^2 + 2a - 3a^2 + 7 - 4a$

1.3 Simplifying Expressions — Worked Solution

Simplify: $5a^2 + 2a - 3a^2 + 7 - 4a$

SOLUTION

$$\begin{aligned} &5a^2 + 2a - 3a^2 + 7 - 4a \\ &(5a^2 - 3a^2) + (2a - 4a) + 7 \\ &2a^2 - 2a + 7 \end{aligned}$$

Answer: $2a^2 - 2a + 7$

1.4 Evaluating Algebraic Expressions

Replace variables with values, then evaluate using order of operations.

1

SUBSTITUTE

Write each given value in place of its variable.

2

PARENTHESES

Use grouping around negative or fractional values.

3

CALCULATE

Evaluate powers and operations after substitution.

1.4 Evaluating Algebraic Expressions

Evaluate $2x^2 - 3y$ when $x = -2$ and $y = 5$.

1.4 Evaluating Algebraic Expressions — Worked Solution

Evaluate $2x^2 - 3y$ when $x = -2$ and $y = 5$.

SOLUTION

$$2(-2)^2 - 3(5)$$

$$2(4) - 15$$

$$8 - 15 = -7$$

Answer: -7

1.4 Evaluating Algebraic Expressions

Use $A = \frac{1}{2}h(b_1 + b_2)$. Find A when $b_1 = 8$, $b_2 = 14$, and $h = 5$.

1.4 Evaluating Algebraic Expressions — Worked Solution

Use $A = \frac{1}{2}h(b_1 + b_2)$. Find A when $b_1 = 8$, $b_2 = 14$, and $h = 5$.

SOLUTION

$$A = \frac{1}{2}(5)(8 + 14)$$

$$A = 2.5(22)$$

$$A = 55$$

Answer: 55 square units

1.5 Distributive Property

The distributive property multiplies one factor by every term in a group.

1 EXPAND

Multiply the factor by every term.

2 NEGATIVE

A negative factor changes every product's sign.

3 FACTOR

Reverse distribution with the greatest common factor.

4 EACH TERM

Multiply the factor by every term.

5 GCF

Use the greatest common factor to factor.

1.5 Distributive Property

Expand: $-6(2x - 5)$

1.5 Distributive Property — Worked Solution

Expand: $-6(2x - 5)$

SOLUTION

$$\begin{aligned} & -6(2x - 5) \\ & (-6)(2x) + (-6)(-5) \\ & -12x + 30 \end{aligned}$$

Answer: $-12x + 30$

1.5 Distributive Property

Factor completely: $7x + 21$

1.5 Distributive Property — Worked Solution

Factor completely: $7x + 21$

SOLUTION

Greatest common factor: 7

$$7x + 21 = 7(x + 3)$$

Check: $7 \cdot x + 7 \cdot 3$

Answer: $7(x + 3)$

1.5 Distributive Property

Simplify: $4(3m - 2) - 2(m + 5)$

1.5 Distributive Property — Worked Solution

Simplify: $4(3m - 2) - 2(m + 5)$

SOLUTION

$$\begin{aligned} &4(3m - 2) - 2(m + 5) \\ &12m - 8 - 2m - 10 \\ &(12m - 2m) + (-8 - 10) \\ &10m - 18 \end{aligned}$$

Answer: $10m - 18$

1.6 Descriptive Modeling and Accuracy

A descriptive model turns quantities and relationships into mathematics. Accuracy means reporting a value at a precision that fits the situation.

1 DEFINE

Name variables; what the represent and include units.

2 RELATE

Translate rates and totals into an expression.

3 INTERPRET

Explain the value in the original situation.

4 CALCULATE

Keep extra digits until final rounding.

5 ROUND

Round the final value to a practical place.

6 CHECK

Test the model with a reasonable value.

1.6 Descriptive Modeling and Accuracy

A taxi charges \$3.50 plus \$2.25 per mile.

Write a model (equation) and find the cost of a 6.4-mile ride.

1.6 Descriptive Modeling and Accuracy — Worked Solution

A taxi charges \$3.50 plus \$2.25 per mile.

Write a model and find the cost of a 6.4-mile ride.

SOLUTION

Let m = miles and C = cost.

$$C = 3.50 + 2.25m$$

$$C = 3.50 + 2.25(6.4)$$

$$C = 17.90$$

**Answer: $C = 3.50 + 2.25m$;
the ride costs \$17.90.**

1.6 Descriptive Modeling and Accuracy

A room is 12.6 ft by 9.8 ft. Flooring is sold by the whole square foot. How many square feet should be purchased?

1.6 Descriptive Modeling and Accuracy — Worked Solution

A room is 12.6 ft by 9.8 ft. Flooring is sold by the whole square foot. How many square feet should be purchased?

SOLUTION

$$\text{Area} = \text{length} \times \text{width}$$

$$A = 12.6(9.8)$$

$$A = 123.48 \text{ ft}^2$$

Round up for full coverage.

Answer: purchase at least 124 ft² of flooring



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